



Eigenshape analysis of ammonoid sutures

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A morphometric method based on eigenshape analysis is proposed for characterizing the morphospace of widely varied ammonoid suture shapes. The analysis requires initially the placement of a suture line in an x - y coordinate system, with the ventral and umbilical extremes located on the x -axis. Series of x - and y -coordinates along the suture line are used as descriptors. The coordinate data are summarized into the two largest principal components using eigenshape analysis. A total of 115 species belonging to six ammonoid orders, spanning from the Devonian to the Cretaceous, was utilized in the present analysis. The analysis, using y -coordinate data, revealed differences in morphological variation in suture shape among taxa within the Mesozoic ammonitids: the Lytoceratina and Ammonitina were characterized by small negative values of the first eigenshape scores, whereas the Phylloceratina (the sister group of the Lytoceratina plus Ammonitina), as well as the Triassic ceratitids and Palaeozoic ammonoids, have a wide range of the first eigenshape scores. The pattern of data obtained from many different ammonoid species, as plotted on eigenshape axes in the morphospace constructed based on y -coordinate data, reveals a plesiomorphic aspect of suture shape in some phylloceratine species with respect to other ammonitids. □ *Ammonoids, eigenshape analysis, morphometrics, suture line.*

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The ammonoid suture has long been a subject of palaeontologists' attention because of its great taxonomic utility (Arkell 1957; Miller *et al.* 1957; Kullmann & Wiedmann 1970; Wiedmann & Kullmann 1981; Korn & Klug 2002; Korn *et al.* 2003; Shevryev 2006), morphogenetic wonder (Seilacher 1975; Westermann 1975; Bayer 1978; García-Ruiz *et al.* 1990; Hewitt *et al.* 1991; Hammer 1999) and puzzling functional morphology (Westermann 1958, 1975; Henderson 1984; Hewitt & Westermann 1986, 1987, 1997; Jacobs 1990; Weitschat & Bandel 1991; Olóriz & Palmqvist 1995; Saunders 1995; Seilacher & LaBarbera 1995; Saunders & Work 1996, 1997; Daniel *et al.* 1997; Olóriz *et al.* 1997; Hassan *et al.* 2002; Lewy 2002; Pérez-Claros 2005; Pérez-Claros *et al.* 2007; De Blasio 2008). The suture is the curved line of intersection between the external shell wall and a series of septa that partition the phragmocone internally into phragmocone chambers. Each suture consists of backward convex or concave folds that are termed lobes and saddles respectively. The lobes are classified into

several categories according to their primary positions. The pattern of their arrangement, as well as nature of its ontogenetic change, is used as a taxonomic characteristic of ammonoids (Arkell 1957; Kullmann & Wiedmann 1970; Lehmann 1981; Wiedmann & Kullmann 1981; Korn & Klug 2002; Korn *et al.* 2003; Shevryev 2006).

Many studies have sought to numerically quantify the suture geometry for the purpose of taxonomic comparison and functional morphological analysis. Most studies have adopted a mathematical representation of suture complexity using simple indices (Westermann 1971; Ward 1980; Jacobs 1990; Batt 1991; Saunders 1995; Saunders & Work 1996, 1997; Saunders *et al.* 1999; Ballantine 2007) or the fractal dimension (García-Ruiz *et al.* 1990; Boyajian & Lutz 1992; Lutz & Boyajian 1995; Olóriz & Palmqvist 1995; Checa & García-Ruiz 1996; Olóriz *et al.* 2002; Pérez-Claros *et al.* 2002; Pérez-Claros 2005); however, such approaches rely upon proxies of suture complexity rather than descriptors of the suture shape itself.

One promising approach to provide a unique definition of suture shape is Fourier- or wavelet-based methods in which a periodic function describing the suture shape is decomposed into frequency and/or location domains (Canfield & Anstey 1981; Gildner 2003; Allen 2006, 2007); however, such an approach generally yields a large number of coefficients in representing the shape of interest, meaning that an additional multivariate analysis is required for the ordination of a population of shapes in the morphospace. A pattern matching technique using geographic information systems (Manship 2004) appears to be useful for the identification of suture patterns, although it provides a measure of the difference between sutures rather than a descriptor of suture shape itself.

In the present study, we propose an alternative approach to the quantitative description of suture shape based on an eigenshape analysis originally developed by Lohmann (1983). This method is designed to characterize the morphospace of a wide variety of ammonoid suture forms. The occupation pattern in the morphospace is compared among taxonomic groups to assess phylogenetic aspects of morphometric data on suture shape.

Materials and methods

Measurements of suture lines

We studied 128 specimens from 115 species belonging to six orders ranging in age from the Devonian to the Cretaceous (Table 1). The higher taxonomy follows Korn & Klug (2002) for Devonian ammonoids, Becker & Kullmann (1996) for Carboniferous to Permian ammonoids, Tozer (1981) for Triassic ammonoids, and Arkell *et al.* (1957) and Wright *et al.* (1996) for Jurassic and Cretaceous ammonitids. All the examined specimens are housed in the University Museum of the University of Tokyo (UMUT), Ohio University (OUZ) or Shizuoka University (SUM).

A piece of external hemi-suture was analysed for each sub-adult individual. To capture an image of the suture line, each portion of the external surface of each specimen (from which the shell wall had been removed) was photographed using a Keyence VH-5000 CCD camera, viewed perpendicular to the specimen surface (Ubukata 2004). Captured images were pieced together to form a synthetic image of the external hemi-suture, using Justsystem Hanako Photo-Retouch software on a personal computer. A series of x - and y -coordinates digitized along the suture line was translated, rotated and scaled in the x - y

coordinate system such that the ventral and umbilical extremes were placed on the x -axis, separated by 10 000 pixels (Fig. 1A). Next, 4096 equally spaced points were interpolated along the series of normalized coordinate points using a cubic spline.

Eigenshape analysis of ammonoid sutures

To summarize the coordinate data into a small number of principal components, a modified version of eigenshape analysis was applied to the descriptors. The eigenshape analysis performs well with a variety of organic shapes that include not only closed outlines but also open curves (MacLeod 1999). The original eigenshape analysis is based on a series of angular deviations between adjacent coordinate points as a function of arc length along the object outline – an approach that is called ‘Zahn and Roskies’ function’ (Zahn & Roskies 1972; Lohmann 1983). However, this function represents differences between adjacent points and is readily distorted by high-frequency noise, especially in the analysis of an intricate curve such as an ammonite suture. Accordingly, in the present case we employed a series of normalized and equally spaced x - and y -coordinate data as descriptors for eigenshape analysis. As a series of x -coordinates is not a periodic function but logically increases with increasing distance from the venter (Fig. 1C), such series were detrended by subtracting the expected linear increase in x to introduce periodicity to the x function (x' ; see Fig. 1D). A series of normalized, equally spaced and/or detrended x' - or y -coordinate data for the k th individual is tentatively notated by z :

$$\mathbf{z}_k = (z_{k,1}, z_{k,2}, \dots, z_{k,4096}).$$

Principal components were computed by singular value decomposition of interobject covariances of the descriptors of each set of z data for the i th point:

$$\mathbf{z}'_i = (z_{1,i}, z_{2,i}, \dots, z_{n,i}),$$

where n is the total number of individuals (i.e. $n = 128$ in the present case). The j th eigenshape ($\omega_{i,j}$) for the i th point was calculated as an inner product between \mathbf{z}'_i and the eigenvector of the covariance matrix (\mathbf{v}_j) divided by the corresponding j th largest singular value (μ_j):

$$\omega_{j,i} = \frac{\mathbf{z}'_i \cdot \mathbf{v}_j}{\mu_j}.$$

A series of the eigenshapes for individual points makes up the j th eigenshape vector:

Table 1. List of examined specimens.

Species	Specimens	Age	Locality
Agoniatitida			
Agoniatitina			
<i>Latanarcestes</i> sp.	UMUT-PM-29049	Devonian	Taouz, Morocco
<i>Fidelites</i> sp.	UMUT-PM-29050	Devonian	Erfoud, Morocco
<i>Achguigites</i> sp.	UMUT-PM-29051	Devonian	Erfoud, Morocco
Gephuroceratina			
<i>Pseudoproboloceras costulatum</i>	UMUT-PM-30079	Devonian	Taouz, Morocco
<i>Beloceras</i> sp.	UMUT-PM-29053	Devonian	Erfoud, Morocco
Anarcestina			
<i>Anarcestes</i> sp.1	UMUT-PM-30080	Devonian	Erfoud, Morocco
<i>Anarcestes</i> sp.2	UMUT-PM-30081	Devonian	Erfoud, Morocco
<i>Praewerneroceras hollardi</i>	Ouz-5600	Devonian	Taouz, Morocco
Pharciceratina			
<i>Stenopharciceras</i> sp.	UMUT-PM-29058	Devonian	Taouz, Morocco
<i>Stenopharciceras lunulicosta</i>	UMUT-PM-30082	Devonian	Taouz, Morocco
<i>Synpharciceras clavilobum</i>	UMUT-PM-30083	Devonian	Taouz, Morocco
Goniatitida			
Tornoceratina			
<i>Epitornoceras mithracoides</i>	UMUT-PM-29060	Devonian	Taouz, Morocco
<i>Epitornoceras mithracoides</i>	UMUT-PM-30084	Devonian	Taouz, Morocco
<i>Phoenixites</i> aff. <i>frechi</i>	UMUT-PM-30085	Devonian	Taouz, Morocco
<i>Cheiloceras undulosum</i>	UMUT-PM-30086	Devonian	Taouz, Morocco
<i>Sporadoceras</i> sp.	UMUT-PM-29064	Devonian	Taouz, Morocco
<i>Sporadoceras muensteri</i>	UMUT-PM-30087	Devonian	Erfoud, Morocco
<i>Imitoceras rotatorium</i>	Ouz-5601	Carboniferous	Rockford, Indiana
Goniatitina			
<i>Girtyoceras meslerianum</i>	Ouz-5602	Carboniferous	Jackforth Creek, Oklahoma
<i>Eumorphoceras bisulcatum</i>	Ouz-5603	Carboniferous	Leslie, Searcy Co., Arkansas
<i>Hudsonoceras proteum</i>	UMUT-PM-30088	Carboniferous	Knockauns Mts, Clare Co., Ireland
<i>Goniatites</i> aff. <i>crenistris</i>	UMUT-PM-29069	Carboniferous	Jackforth Creek, Oklahoma
<i>Goniatites multiliratum</i>	UMUT-PM-29070	Carboniferous	Jackforth Creek, Oklahoma
<i>Perrinites hilli</i>	UMUT-PM-30089	Permian	Las Pelicias, Coahuila, Mexico
<i>Cravenoceras hesperium</i>	Ouz-5604	Carboniferous	Death Valley, California
<i>Neodimorphoceras</i> sp.	Ouz-5605	Carboniferous	Texas
<i>Cymoceras</i> sp.	Ouz-5606	Carboniferous	Fayetteville, Washington Co., Arkansas
<i>Glaphyrites clinei</i>	UMUT-PM-30090	Carboniferous	Collinsville, Oklahoma
<i>Syngastrioceras oblatum</i>	Ouz-5607	Carboniferous	Fayetteville, Washington Co., Arkansas
<i>Homoceras smithi</i>	Ouz-5608	Carboniferous	Knockauns Mts, Clare Co., Ireland
<i>Bisatoceras</i> sp.	UMUT-PM-30091	Carboniferous	Oklahoma
<i>Bisatoceras primum</i>	Ouz-5609	Carboniferous	Oklahoma
<i>Thalassoceras gemmellari</i>	UMUT-PM-29078	Permian	Actasty R., S. Ural, Kazakhstan
<i>Pseudoparalegoceras kesslerense</i>	Ouz-5610	Carboniferous	Winslow, Washington Co., Arkansas
<i>Wellerites mohri</i>	Ouz-5611	Carboniferous	Carroll, Ohio
<i>Gonioloboceras</i> sp.	Ouz-5612	Carboniferous	South Bend, Texas
<i>Mescalites</i> sp.	Ouz-5613	Permian	Tularosa, New Mexico
<i>Wewokites</i> sp.	UMUT-PM-30092	Carboniferous	Oklahoma
<i>Crimites subkrotowi</i>	UMUT-PM-30093	Permian	Actasty R., S. Ural, Kazakhstan
<i>Peritrochia typicus</i>	UMUT-PM-29080	Permian	Actasty R., S. Ural, Kazakhstan
<i>Peritrochia typicus</i>	UMUT-PM-30094	Permian	Actasty R., S. Ural, Kazakhstan
<i>Peritrochia invaribilis</i>	UMUT-PM-30095	Permian	Actasty R., S. Ural, Kazakhstan
<i>Uraloceras involutum</i>	UMUT-PM-30096	Permian	Actasty R., S. Ural, Kazakhstan
<i>Popanoceras annae</i>	UMUT-PM-30097	Permian	Actasty R., S. Ural, Kazakhstan
Clymeniida			
Cyrtoclymeniina			
<i>Cymaclymenia</i> sp.1	UMUT-PM-29089	Devonian	Morocco
<i>Cymaclymenia</i> sp.2	UMUT-PM-29090	Devonian	Morocco
Clymeniina			
<i>Platyclymenia</i> sp.1	UMUT-PM-29091	Devonian	Morocco
<i>Platyclymenia</i> sp.2	UMUT-PM-29092	Devonian	Morocco
<i>Oxyclymenia</i> sp.	UMUT-PM-29094	Devonian	Morocco
<i>Gonioclymenia</i> sp.	UMUT-PM-30098	Devonian	Erfoud, Morocco
Prolecanitida			
<i>Boesites</i> sp.	UMUT-PM-30099	Carboniferous	Rochelle, Texas
<i>Daraelites elegans</i>	UMUT-PM-29097	Permian	Actasty R., S. Ural, Kazakhstan
<i>Akmilleria electraensis</i>	UMUT-PM-29098	Permian	White Pine Co., Nevada
<i>Medlicottia intermedia</i>	UMUT-PM-29099	Permian	Actasty R., S. Ural, Kazakhstan
<i>Neopronorites skvorzovi</i>	UMUT-PM-29100	Permian	Actasty R., S. Ural, Kazakhstan
<i>Pseudopronorites arkansiensis</i>	Ouz-5614	Carboniferous	Woolsey, Arkansas

Table 1. (Continued).

Species	Specimens	Age	Locality
Ceratitida			
<i>Xenoceltites subevolutus</i>	UMUT-MM-29103	Triassic	Spitsbergen, Norway
<i>Xenoceltites subevolutus</i>	UMUT-MM-30100	Triassic	Wallenbergfjellet, Spitsbergen, Norway
<i>Paraceltites elegans</i>	UMUT-PM-29101	Permian	Gaudalupe Mts., Texas
<i>Dinartes asiaticus</i>	UMUT-MM-30101	Triassic	Mangyshlak, Dolnaya, Kazakhstan
<i>Pseudosagoceras</i> sp.	UMUT-MM-29104	Triassic	Spitsbergen, Norway
<i>Amphipopanoceras</i> cf. <i>medium</i>	UMUT-MM-30102	Triassic	Spitsbergen, Norway
<i>Paranannites spathi</i>	UMUT-MM-29106	Triassic	Crienden Spring, Nevada
<i>Paranannites spathi</i>	UMUT-MM-30103	Triassic	Stensiö-Fiellet, Spitsbergen, Norway
<i>Paranannites aspenensis</i>	UMUT-MM-30104	Triassic	Crienden Spring, Nevada
<i>Prosphingites czezanowski</i>	UMUT-MM-30105	Triassic	Olenek River, Mengilach, Russia
<i>Meekoceras graciliatus</i>	UMUT-MM-30106	Triassic	Crienden Spring, Nevada
<i>Boreomeekoceras keyserlingi</i>	UMUT-MM-30107	Triassic	Olenek River, Mengilach, Russia
<i>Arctoprioceras nodosus</i>	UMUT-MM-30108	Triassic	Stensiö-Fiellet, Spitsbergen, Norway
<i>Dieneroceras spathi</i>	UMUT-MM-30109	Triassic	Crienden Spring, Nevada
<i>Arctoceras blomstrandii</i>	UMUT-MM-30110	Triassic	Spitsbergen, Norway
<i>Nordophraceras schmidtii</i>	UMUT-MM-30111	Triassic	Olenek River, Mengilach, Russia
<i>Wasatchites tridentinus</i>	UMUT-MM-30112	Triassic	Botheheia, Spitsbergen, Norway
<i>Wasatchites tardus</i>	UMUT-MM-30113	Triassic	Spitsbergen, Norway
<i>Ceratites nodosus</i>	UMUT-MM-30114	Triassic	Würzburg, Germany
<i>Anagymnoceras varium</i>	UMUT-MM-30115	Triassic	Wallenbergfjellet, Spitsbergen, Norway
<i>Favreticeras wallacei</i>	UMUT-MM-29117	Triassic	McCoy Mine, Nevada
<i>Stolleyites tenuis</i>	UMUT-MM-29119	Triassic	Spitsbergen, Norway
<i>Olenekoceras middendorffi</i>	UMUT-MM-30116	Triassic	Olenek River, Mengilach, Russia
<i>Olenikites spiniplicatus</i>	UMUT-PM-30117	Triassic	Olenek River, Mengilach, Russia
<i>Svalbardiceras spitsbergense</i>	UMUT-PM-30118	Triassic	Wallenbergfjellet, Spitsbergen, Norway
<i>Arctohungarites trififormis</i>	UMUT-PM-30119	Triassic	Olenek Bay, Laptev Sea, Russia
<i>Lenotropites caurus</i>	UMUT-PM-30120	Triassic	West Humboldt Range, Nevada
<i>Pseudosvalbardiceras sibiricum</i>	UMUT-PM-30121	Triassic	Olenek River, Mengilach, Russia
Ammonitida			
Phylloceratina			
<i>Phylloceras consanguineans</i>	UMUT-MM-29121	Jurassic	Sakaraha, Madagascar
<i>Phylloceras</i> sp.	UMUT-MM-29122	Cretaceous	Mahajang, Madagascar
<i>Holcophylloceras</i> sp.	UMUT-MM-29123	Jurassic	Sakaraha, Madagascar
<i>Calliphylloceras</i> sp.	UMUT-MM-29124	Jurassic	Sakaraha, Madagascar
<i>Ptychophylloceras</i> sp.	UMUT-MM-29125	Jurassic	Sakaraha, Madagascar
<i>Phyllopachyceras ezoense</i>	UMUT-MM-29126	Cretaceous	Saku, Hokkaido, Japan
<i>Phyllopachyceras ezoense</i>	UMUT-PM-30122	Cretaceous	Saku, Hokkaido, Japan
<i>Hypophylloceras subramosum</i>	UMUT-PM-30123	Cretaceous	Saku, Hokkaido, Japan
<i>Hypophylloceras subramosum</i>	UMUT-PM-30124	Cretaceous	Saku, Hokkaido, Japan
<i>Tragophylloceras ibex</i>	UMUT-MM-29130	Jurassic	Osuabrük, Germany
Lytoceratina			
<i>Pterolytoceras</i> sp.	UMUT-MM-29131	Jurassic	Sakaraha, Madagascar
<i>Argonauticeras</i> sp.	UMUT-MM-29132	Cretaceous	Mahajang, Madagascar
<i>Tetragonites glabrus</i>	UMUT-PM-30125	Cretaceous	Saku, Hokkaido, Japan
<i>Tetragonites glabrus</i>	UMUT-MM-29134	Cretaceous	Tappu, Hokkaido, Japan
<i>Tetragonites popetensis</i>	UMUT-PM-30126	Cretaceous	Saku, Hokkaido, Japan
<i>Eotetragonites</i> sp.	UMUT-MM-29136	Cretaceous	Mahajang, Madagascar
<i>Gaudryceras striatum</i>	SUM-RC-MM004	Cretaceous	Saku, Hokkaido, Japan
<i>Gaudryceras striatum</i>	UMUT-PM-30127	Cretaceous	Saku, Hokkaido, Japan
<i>Gaudryceras tenuiliratum</i>	UMUT-PM-30128	Cretaceous	Saku, Hokkaido, Japan
<i>Gaudryceras</i> sp.	UMUT-PM-30129	Cretaceous	Saku, Hokkaido, Japan
<i>Gaudryceras</i> sp.	UMUT-PM-30130	Cretaceous	Saku, Hokkaido, Japan
Ammonitina			
<i>Grammoceras doerntense</i>	UMUT-MM-29147	Jurassic	Döruten, Germany
<i>Hecticoceras</i> sp.	SUM-RC-MM020	Jurassic	Saltwick Nab, Yorkshire, England
<i>Taramelliceras</i> sp.	UMUT-MM-29149	Jurassic	Sakaraha, Madagascar
<i>Lissoceras</i> sp.	UMUT-MM-29150	Jurassic	Sakaraha, Madagascar
<i>Grossouvria</i> sp.	UMUT-MM-29152	Jurassic	Sakaraha, Madagascar
<i>Aspidoceras</i> sp.	UMUT-MM-29153	Jurassic	Sakaraha, Madagascar
<i>Euaspidoceras</i> sp.	UMUT-MM-29154	Jurassic	Sakaraha, Madagascar
<i>Craspedites subditus</i>	UMUT-PM-30131	Jurassic	Iwanowa, Russia
<i>Desmoceras latidorsatum</i>	UMUT-MM-29156	Cretaceous	Mahajang, Madagascar
<i>Tragodesmoceroideis subcostatus</i>	UMUT-MM-29157	Cretaceous	Tappu, Hokkaido, Japan
<i>Damesites semicostatus</i>	UMUT-PM-30132	Cretaceous	Kotanbetsu, Hokkaido, Japan
<i>Damesites</i> sp.	UMUT-MM-29158	Cretaceous	Saku, Hokkaido, Japan
<i>Hauericeras angustum</i>	SUM-RC-MM006	Cretaceous	Saku, Hokkaido, Japan
<i>Hauericeras angustum</i>	UMUT-PM-30135	Cretaceous	Tappu, Hokkaido, Japan

Table 1. (Continued).

Species	Specimens	Age	Locality
<i>Puzosia</i> sp.	UMUT-MM-29162	Cretaceous	Mahajang, Madagascar
<i>Yokoyamaoceras ishikawai</i>	UMUT-MM-29164	Cretaceous	Saku, Hokkaido, Japan
<i>Yokoyamaoceras ishikawai</i>	UMUT-PM-30136	Cretaceous	Saku, Hokkaido, Japan
<i>Cleonicerias besairiei</i>	UMUT-MM-29167	Cretaceous	Mahajang, Madagascar
<i>Anapachydiscus naumanni</i>	UMUT-MM-29168	Cretaceous	Saku, Hokkaido, Japan
<i>Teshioites ryugasensis</i>	UMUT-MM-29169	Cretaceous	Saku, Hokkaido, Japan
<i>Canadoceras kosmatti</i>	UMUT-PM-30133	Cretaceous	Saku, Hokkaido, Japan
<i>Canadoceras kosmatti</i>	UMUT-PM-30134	Cretaceous	Saku, Hokkaido, Japan
<i>Neogastrolites muelleri</i>	UMUT-MM-29171	Cretaceous	Teigen, Petroleum Co., Montana

$$\omega_j = (\omega_{j,1}, \omega_{j,2}, \dots, \omega_{j,4096}).$$

The covariance between the j th eigenshape vector (ω_j) and each shape function (z_k) defines the j th eigenshape score (=the principal component score) of each suture shape. All specimens listed in Table 1 were analysed together to obtain the first and second eigenshape scores.

Synthetic models of the x' and y functions can be generated along each eigenshape axis to visualize the variation in each component. Synthetic models for the

first (E_1) and second (E_2) eigenshape scores are, respectively, given by

$$z_1^* = E_1 \omega_1$$

and

$$z_2^* = \bar{z}_1^* + E_2 \omega_2.$$

Synthetic models along eigenshape axes were reproduced to assist in the geometric interpretation of the analytical results.

Results

The eigenshape scores obtained using all of the studied specimens are plotted in Figures 2, 3, with the data subdivided into the following three groups: agoniatitids, goniatitids and clymeniids (Figs 2A, 3A), prolecanitids and ceratitids (Figs 2B, 3B) and ammonitids (Figs 2C, 3C).

Analysis of the y function

The first and second eigenshapes (ES1 and ES2) account for 34.6% and 20.9% of the total variance respectively. Synthetic models of the y function reconstructed along the eigenshape axes indicate that both ES1 and ES2 reflect the difference among individuals of the shape of the first-order suture elements (Fig. 2D). As E_1 proceeds in the positive direction along the ES1 axis, the left-hand portion of the synthetic model becomes strongly concaved (E_1 maximum in Fig. 2D); i.e. a wide lobe tends to develop in the ventro-lateral area of the shell. By contrast, an individual with a negative E_1 tends to form a prominent saddle in the ventro-lateral region (E_1 minimum in Fig. 2D), as typically observed in the *Lytoceras* and *Ammonitina*. Positive perturbations associated with ES2 represent the development of a wide lateral lobe lying slightly toward the venter (E_2 maximum in Fig. 2D), whereas the same portion is dominated by a

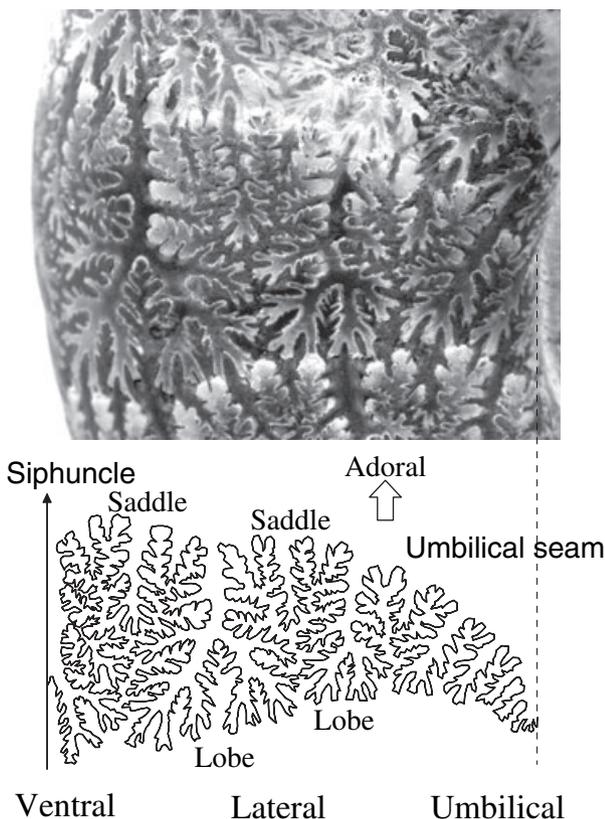


Fig. 1. A lateral view of an ammonoid whorl and trace of a suture line showing terminology on major suture elements in relation to the parts of the shell.

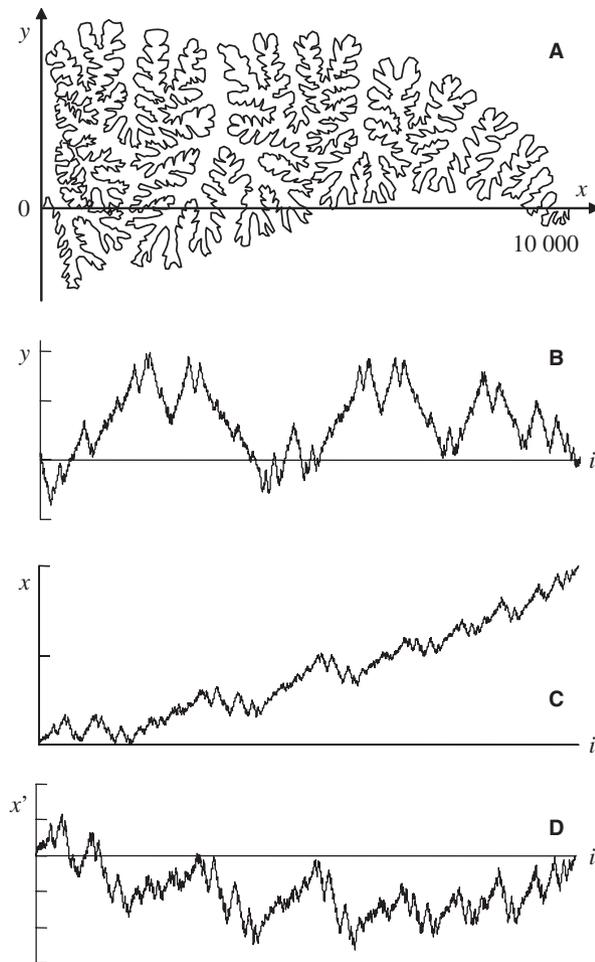


Fig. 2. Measurements of coordinate data along a suture line. A digitized suture line was placed in a reference system such that its ventral and umbilical extremes were located on the x -axis, separated by 10 000 pixels (A). A series of y -coordinate values measured along the suture line represents a periodic function of the cumulative chordal length (l) of the suture line (B). As a series of x -coordinate values is not periodic but logically increases (C), such series were detrended by subtracting the expected linear increase in x to introduce periodicity to the x function (D).

prominent saddle when E_2 is negative (E_2 minimum in Fig. 2D).

Plots of E_2 versus E_1 are effective in depicting differences in the y function among taxa (Fig. 2A–C). Most species belonging to the Agoniatitida, Clymeniida or Prolecanitida occupy the region around the origin of the morphospace (Fig. 2A, B), even though the suture shape appears dissimilar among these taxa. The mean shape of synthetic models of the y function illustrated at the origin of the morphospace is approximately flat, with little amplitude (see ‘mean’ in Fig. 2D); i.e. suture shapes surrounding the origin are characterized by small amplitude regardless of the number of the first-order elements. By contrast, both goniatitids and ceratitids cover a wider area of the morphospace (Fig. 2A, B). In the Mesozoic ammonitids, most

species belonging to the Lytoceratina and Ammonitina are characterized by a small negative value of E_1 , which represents the prominence of the ventrally situated saddle; however, the Phylloceratina, in addition to the goniatitids and ceratitids, cover a wide range of E_1 (Fig. 2C). With regard to E_2 , all individuals belonging to the Phylloceratina have positive E_2 . Most phylloceratine species have positive E_1 and/or large E_2 ; this condition indicates that the largest lateral lobe is located at the ventro-lateral portion of the shell.

Analysis of the x' function

The $ES1$ is outstanding among eigenshapes, and $ES1$ and $ES2$ explain 57.3% and 9.4% of the total variance respectively. Synthetic models of the x' function reconstructed along the eigenshape axes clearly show the close relationship between eigenshape scores and concavity of the model (Fig. 3D). Positive perturbations associated with each eigenshape axis represent the strongly concave shape of the synthetic model; however, the position of the trough in the model is different between large E_1 and large E_2 extremes: it lies toward the umbilicus in the former case.

The distribution of eigenshape scores is different between ammonitids and other ammonoids (Fig. 3A–C): agoniatitids, goniatitids, clymeniids, prolecanitids and ceratitids cover a wide range from the negative to positive E_1 regions of the morphospace, although all have E_1 values below 450 (Fig. 3A, B); by contrast, the Ammonitida occupy the large E_1 area. In particular, the Phylloceratina possess E_1 values greater than 420 (Fig. 3C). This distribution reflects the strongly concave shape of the x' function, as characteristically found in ammonitids.

Discussion

In the context of ammonoid macrotaxonomy, primary sutures – the fundamental sutures formed at the early ontogeny – have attracted greater attention than have adult sutures. The number of primary suture elements, as well as the manner of subsequent introduction of new lobes, has been regarded as an important criterion in subdividing ammonoids into several orders and/or suborders (Schindewolf 1954; Kullmann & Wiedmann 1970; Wiedmann & Kullmann 1981; Korn *et al.* 2003). Comparative anatomy and relevant nomenclature on ammonoid sutures are based on their ontogenetic development rather than geometric similarity. Consequently, the geometric properties of adult sutures have principally inspired studies of functional morphology (Henderson 1984; Jacobs 1990; Daniel *et al.* 1997; Hewitt &

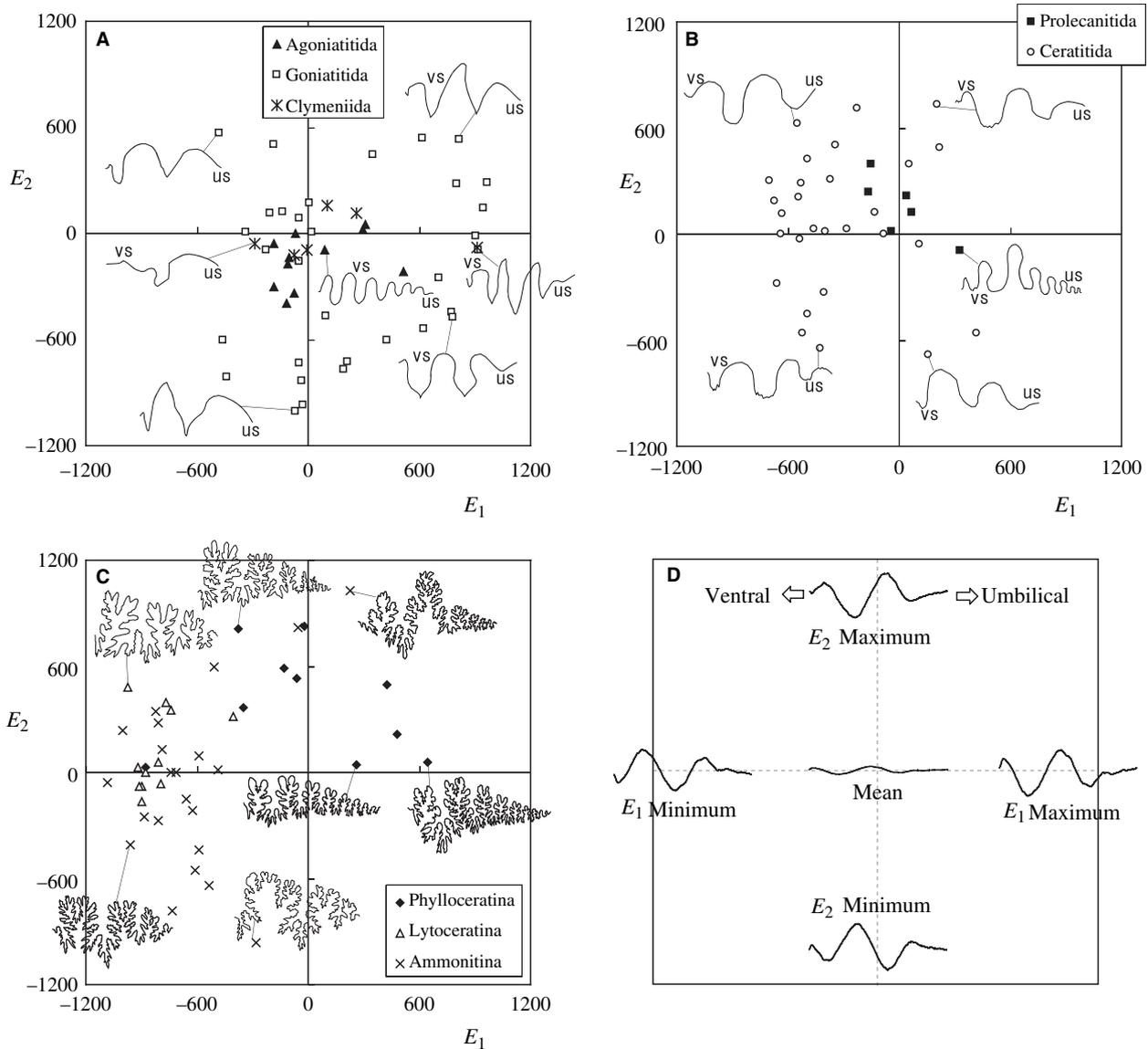


Fig. 3. Results of eigenshape analysis based on the y function. A–C, scatter plots of eigenshape scores on the first two eigenshapes, showing examples of selected suture shapes. The position of the ventro-lateral shoulder (vs) and umbilical shoulder (us) are shown in each diagram. A, Agoniatitida, Goniatitida and Clymeniida. B, Prolecanitida and Ceratitida. C, Ammonitida. D, synthetic models visualizing the shape of the y function at the extremes of the first two eigenshape axes (maximum and minimum) and for the mean ES1 score.

Westermann 1997; Olóriz *et al.* 1997; Saunders & Work 1997; Hassan *et al.* 2002; Lewy 2002; Pérez-Claros *et al.* 2007) rather than assessments of phylogenetic aspects. The present study revealed differences in occupation pattern in the morphospace of adult sutures among higher taxa of ammonoids.

Analysis of the y function revealed that goniatitids and ceratitids cover a wide region of the morphospace (Fig. 2A, B). The area in the morphospace covered by each taxon depends on how many species were sampled; a much larger number of goniatitid and ceratitid species were utilized in this study than were agoniatitids, clymeniids and prolecanitids. Therefore, the result does not necessarily indicate that the Goniatitida and

Ceratitida are more disparate than the other Palaeozoic orders, although it does show that both the goniatitid and ceratitid samples accommodate a wide variation in suture shapes surrounding the origin of the morphospace.

By contrast, lytoceratine and ammonitine species have negative E_1 values that are much smaller in value than those of other ammonoids (Fig. 2). The typical lytoceratine and ammonitine sutures develop a prominent saddle near the venter; this feature is less common in other ammonoids (Figs 2, 4). However, the Phylloceratina – the stem group of the Ammonitida – have a wide range in E_1 , as do the Triassic ceratitids and Palaeozoic goniatitids (Fig. 2C). In most

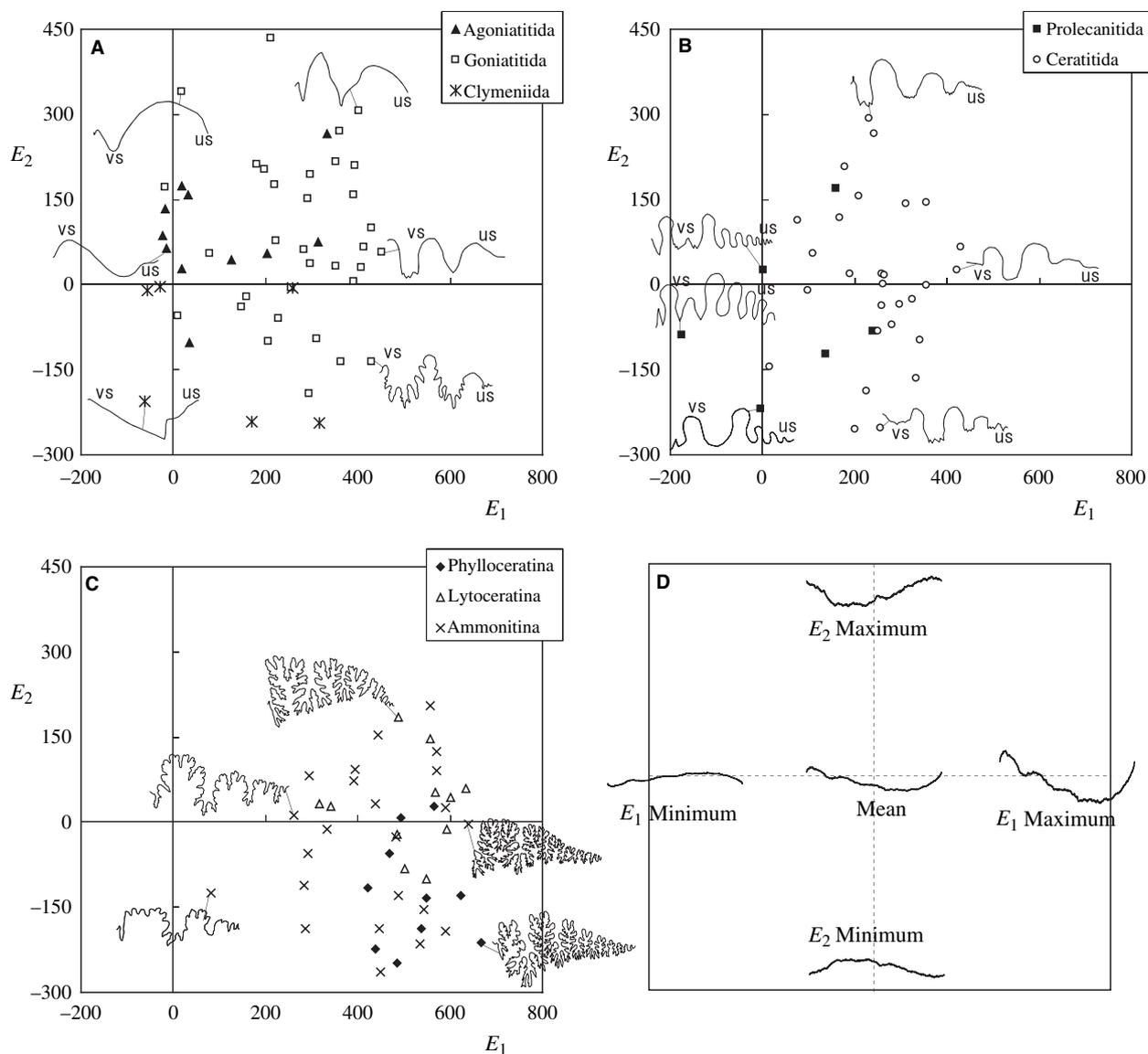


Fig. 4. Results of eigenshape analysis based on the x' function. A–C, scatter plots of eigenshape scores on the first two eigenshapes showing examples of selected suture shapes. The position of the ventro-lateral shoulder (vs) and umbilical shoulder (us) are shown in each diagram. A, Agoniatitida, Goniatitida and Clymeniida. B, Prolecanitida and Ceratitida. C, Ammonitida. D, synthetic models visualizing the shape of the y function at the extremes of the first two eigenshape axes (maximum and minimum) and for the mean ES_1 score.

phylloceratine sutures, the largest saddle is found on the lateral flank of the whorl (Fig. 4), whereas it is situated near the venter in the typical lytoceratine and ammonitine sutures. Given that the Ammonitida are derived from the Ceratitida via some phylloceratine species (Shigeta & Weitschat 2004), the result of the eigenshape analysis for the y function seems to reveal a plesiomorphic aspect of adult suture shapes in some phylloceratine species with respect to other ammonitids that possess more highly specialized suture shapes.

Focusing on the x' function, the phylogenetic aspect of the phylloceratine suture appears to contradict the finding obtained from the analysis of the y function.

Among the examined ammonoid taxa, ammonitid species (particularly phylloceratine ones) tend to occupy an area near the lower right (large E_1 and small E_2) corner of the morphospace (Fig. 2C), whereas the other ammonoids range from the central region to the left-hand side of the diagram shown in Fig. 2A, B. Therefore, the result of the analysis based on the x' function indicates an apomorphic aspect of phylloceratine sutures; however, unlike the y function, any geometric interpretation based on the x' function is not straightforward. The descriptor of the x' function represents the deviation of the observed x -coordinate of each point from the expected x value of the point, which shows a linear increase.

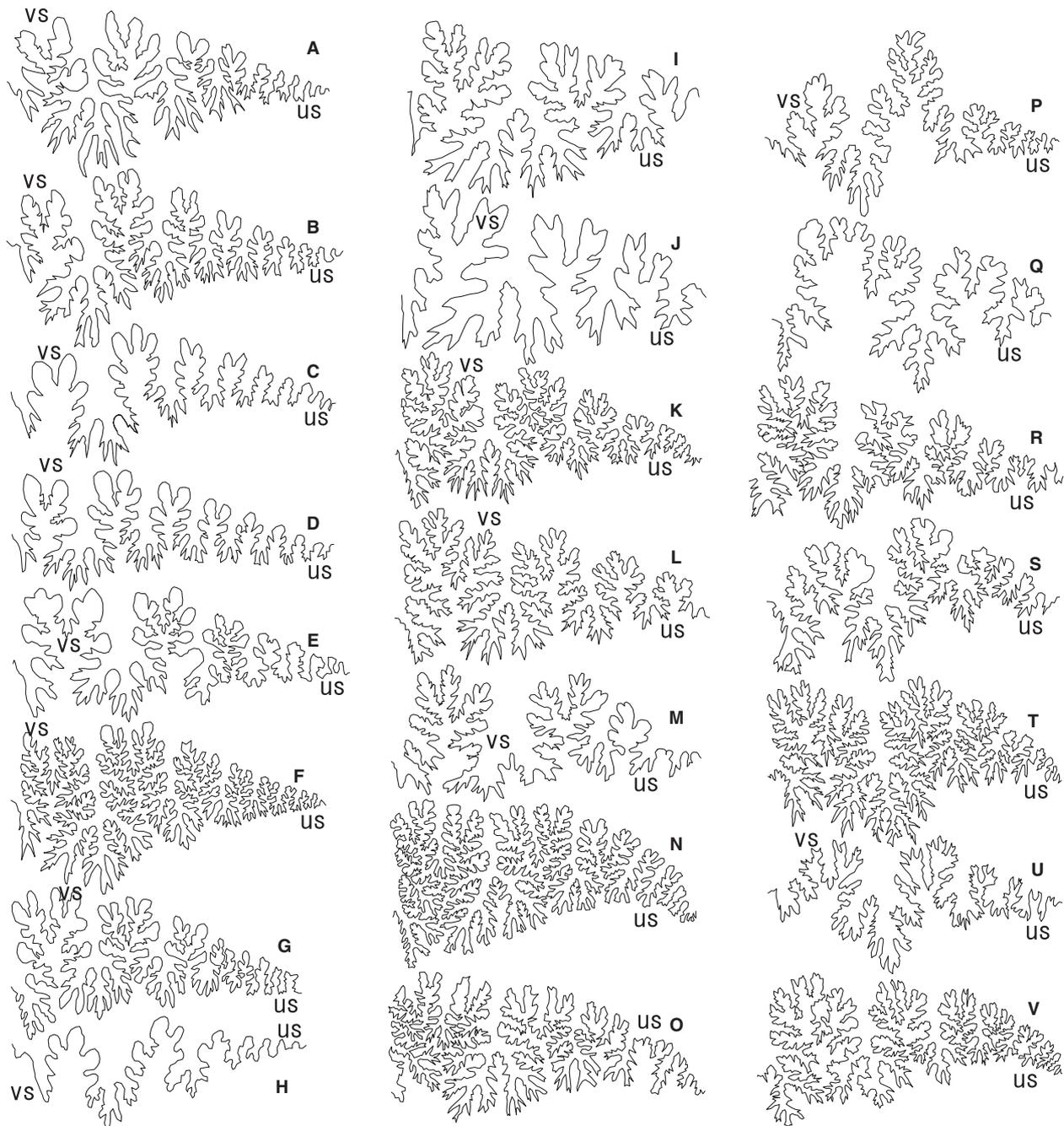


Fig. 5. Traced suture lines in selected ammonitid species. The position of the ventro-lateral shoulder (vs) and umbilical shoulder (us) are shown in each diagram. A, *Phylloceras consanguineum*. B, *Phylloceras* sp. C, *Holcophylloceras* sp. D, *Calliphylloceras* sp. E, *Ptychophylloceras* sp. F, *Hypophylloceras subramosum*. G, *Phyllopachyceras ezoense*. H, *Tragophylloceras ibex*. I, *Pterolytoceras* sp. J, *Argonauticeras* sp. K, *Tetragonites popetensis*. L, *Tetragonites glabrus*. M, *Eotetragonites* sp. N, *Gaudryceras striatum*. O, *Gaudryceras tenuiliratum*. P, *Lissoceras* sp. Q, *Euaspidoceras* sp. R, *Desmoceras latidorsatum*. S, *Hauericeras angustum*. T, *Yokoyamaoceras ishikawai*. U, *Cleoniceras besairiei*. V, *Canadoceras kosmatti*.

To illustrate the nature of the x' function, consider the case of two travellers making their way from the ventral extreme of a suture line to the umbilical extreme via the following different routes: one travelling along the suture line and the other travelling in a straight line. Each travels at a constant speed along the respective route, but both start and arrive at the same

time. At a given time point, the former lags behind or moves ahead of the latter in travelling toward the umbilicus: the x' function represents, as it were, the extent to which the former precedes the latter. Therefore, the former lags behind the latter in the concave portion of the model of the x' function, whereas it precedes the latter in the convex portion. The shape of the

model x' function typically observed in ammonitids is characterized by a wide and deep concavity (Fig. 3C, D), thereby indicating that when travelling along the typical ammonitid suture, one slowly approaches the umbilicus in the ventral to lateral portions but accelerates in the umbilical area. This situation is realized when the suture shape is strongly meandering and when the largest suture element develops in the ventrolateral portion rather than near the umbilicus.

This geometric interpretation of the x' function is concordant with the following interpretation based on the y function: the lytoceratine and ammonitine sutures are characterized by a prominent saddle situated near the venter, whereas for phylloceratine sutures the largest lobe tends to occur in this position. In addition, a widely concave model of the x' function is generated if the amplitude of the meandering suture shows a rapid decline near the umbilicus. Indeed, most phylloceratine species develop lobe splitting near the umbilical seam or auxiliary lobes (Arkell *et al.* 1957), which rapidly reduce their amplitude and sinuosity approaching the umbilicus (Fig. 4). It is likely that this feature of phylloceratine sutures results in their models of the x' function having the most strongly concave shapes (Fig. 5), yielding the highest E_1 values among other ammonoids.

In summary, the present eigenshape analyses captured the following differences in adult suture shape between ammonitids and other ammonoids. In comparison with non-ammonitid species, most lytoceratine and ammonitine species developed the largest saddle near the venter, such that the centre of the density or geometric 'centroid' of the suture curve is ventrally situated; however, such an evolutionary novelty was not achieved by the Phylloceratina, a rootstock of other ammonitids. Instead, the largest lobe of the Phylloceratina is located near the venter, as commonly found in ceratitids and Palaeozoic ammonoids. Nevertheless, the Phylloceratina developed complicated ammonitic sutures, as did the Lytoceratina and Ammonitina, and their first-order suture elements are heavily frilled. However, a series of auxiliary elements of phylloceratine sutures, formed by the splitting of an umbilical lobe and typically seen in prolecanitids and some ceratitids, is much less denticulate than the lateral saddles and lobes, meaning that the 'centroid' of the suture curve is located toward the venter.

To summarize the above results and interpretations, the characteristics of ammonitid sutures are not only their conspicuous complexity but also the ventrally situated position of the 'centroid' of the suture curve. Most phylloceratine species seem to have obtained these characteristics in such way that they partly followed the phylogenetic legacy from ancestral ammonoids; however, more derived lytoceratine and ammonitine

ammonoids appear to have obtained these characteristics via a more drastic reform. A diagnostic sutural characteristic, appropriate for distinction of the Lytoceratina and Ammonitina from the Phylloceratina, is the incised dorsal lobe (Kullmann & Wiedmann 1970; Shevyrev 2006), rather than features of the external suture. Lobe splitting near the umbilical seam, as seen typically in the Phylloceratina, is also found in the Lytoceratina and Ammonitina (Kullmann & Wiedmann 1970; Wiedmann & Kullmann 1981). Nevertheless, the present analysis indicates that differences in morphological variation can be found among taxa even if the shapes of the external sutures are compared using mature individuals. The several derived characteristics of suture shape that are found in the Lytoceratina and Ammonitina support the phylogenetic hypothesis that they are derivatives of the Phylloceratina.

The eigenshape-based method introduced above provides a useful tool in defining a morphospace, which accommodates various suture shapes. This paper presents a case study involving comparisons among higher taxa; however, our approach is also applicable to the comparisons among closely related species and to analyses of within-species variation. The present analysis successfully distilled the first-order morphological components from complex suture curves, but we did not address variations in finer structures. Further refinements will expand the utility of our approach for capturing richer structures and the ordination of various shapes in a variety of organisms.

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References

- Allen, E.G. 2006: New approaches to Fourier analysis of ammonoid sutures and other complex, open curves. *Paleobiology* 32, 299–315.
- Allen, E.G. 2007: Understanding ammonoid sutures: new insight into the dynamic evolution of Paleozoic suture morphology. In Landmann, N.H., Davis, R.A. & Mapes, R.H. (eds.): *Cephalopods Present and Past: New Insights and Fresh Perspectives*, 159–180. Springer-Verlag, Dordrecht.
- Arkell, W.J. 1957: Introduction to Mesozoic Ammonoidea. In Moore, R.C. (ed.): *Treatise on Invertebrate Paleontology, Volume L*, 81–129. Geological Society of America, Boulder, and University of Kansas Press, Lawrence.
- Arkell, W.J., Kummel, B. & Wright, C.W. 1957: Systematic descriptions. In Moore, R.C. (ed.): *Treatise on Invertebrate Paleontology, Volume L*, 129–437. Geological Society of America, Boulder, and University of Kansas Press, Lawrence.
- Ballantine, C.M. 2007: A mathematical analysis of some indices used to classify ammonite shells. *Lethaia* 40, 197–198.
- Batt, R.J. 1991: Sutural amplitude of ammonite shells as a paleoenvironmental indicator. *Lethaia* 24, 219–225.
- Bayer, U. 1978: The impossibility of inverted suture lines in ammonites. *Lethaia* 11, 307–313.

- Becker, R.T. & Kullmann, J. 1996: Paleozoic ammonoids in space and time. In Landman, N., Tanabe, K. & Davis, R.A. (eds): *Ammonoid Paleobiology*, 711–753. Prenum, New York.
- Boyajian, G. & Lutz, T. 1992: Evolution of biological complexity and its relation to taxonomic longevity in the Ammonoidea. *Geology* 20, 983–986.
- Canfield, D.J. & Anstey, R.L. 1981: Harmonic analysis of cephalopod suture patterns. *Mathematical Geology* 13, 23–35.
- Checa, A.G. & García-Ruiz, J.M. 1996: Morphogenesis of the septum in ammonoids. In Landman, N., Tanabe, K. & Davis, R.A. (eds): *Ammonoid Paleobiology*, 253–296. Prenum, New York.
- Daniel, T.L., Helmuth, B.S., Saunders, W.B. & Ward, P.D. 1997: Septal complexity in ammonoid cephalopods increased mechanical risk and limited depth. *Paleobiology* 23, 470–481.
- De Blasio, F.V. 2008: The role of suture complexity in diminishing strain and stress in ammonoid phragmocones. *Lethaia* 41, 15–24.
- García-Ruiz, J.M., Checa, A. & Rivas, P. 1990: On the origin of ammonite sutures. *Paleobiology* 16, 349–354.
- Gildner, R.F. 2003: A Fourier method to describe and compare suture patterns. *Palaeontologia Electronica* 6, 12 <http://palaeo-electronica.org/palaeo/2003_1/suture/issue1_03.htm>.
- Hammer, Ø. 1999: The development of ammonoid septa: an epithelial invagination process controlled by morphogens? *Historical Biology* 13, 153–171.
- Hassan, M.A., Westermann, G.E.G., Hewitt, R.A. & Dokainish, M.A. 2002: Finite-element analysis of simulated ammonoid septa (extinct Cephalopoda): septal and sutural complexities do not reduce strength. *Paleobiology* 28, 113–126.
- Henderson, R.A. 1984: A muscle attachment proposal for septal function in Mesozoic ammonites. *Palaeontology* 27, 461–486.
- Hewitt, R.A. & Westermann, G.E.G. 1986: Function of complexly fluted septa in ammonoid shells. I. Mechanical principles and functional models. *Neues Jahrbuch für Geologie und Paläontologie, Abhandlungen* 172, 47–69.
- Hewitt, R.A. & Westermann, G.E.G. 1987: Function of complexly fluted septa in ammonoid shells. II. Septal evolution and conclusions. *Neues Jahrbuch für Geologie und Paläontologie, Abhandlungen* 174, 135–169.
- Hewitt, R.A. & Westermann, G.E.G. 1997: Mechanical significance of ammonoid septa with complex sutures. *Lethaia* 30, 205–212.
- Hewitt, R.A., Checa, A., Westermann, G.E.G. & Zaborski, P.M. 1991: Chamber growth in ammonites inferred from colour markings and naturally etched surfaces of Cretaceous vascoceratids from Nigeria. *Lethaia* 24, 271–287.
- Jacobs, D.K. 1990: Sutural pattern and shell stress in *Baculites* with implications for other cephalopod shell morphologies. *Paleobiology* 16, 336–348.
- Korn, D. & Klug, C. 2002: Ammonoidea Devonicae. In Riegraf, W. (ed.): *Fossilium Catalogus, Animalia I*, 1–375. Backhuys, Leiden.
- Korn, D., Ebbighausen, V., Bockwinkel, J. & Klug, C. 2003: The A-mode sutural ontogeny in prolecanitid ammonoids. *Lethaia* 46, 1123–1132.
- Kullmann, J. & Wiedmann, J. 1970: Significance of sutures in phylogeny of ammonoidea. *The University of Kansas, Paleontological Contributions* 47, 1–32.
- Lehmann, U. 1981: *The Ammonites: Their Life and Their World*, 246 pp. Cambridge University Press, Cambridge.
- Lewy, Z. 2002: The function of the ammonite fluted septal margins. *Journal of Paleontology* 76, 63–69.
- Lohmann, G.P. 1983: Eigenshape analysis of microfossils: a general morphometric procedure for describing changes in shape. *Mathematical Geology* 15, 659–672.
- Lutz, T.M. & Boyajian, G.E. 1995: Fractal geometry of ammonoid sutures. *Paleobiology* 21, 329–342.
- MacLeod, N. 1999: Generalizing and extending the eigenshape method of shape space visualization and analysis. *Paleobiology* 25, 107–138.
- Manship, L.L. 2004: Pattern matching: classification of ammonitic sutures using GIS. *Palaeontologia Electronica* 7, 15 <http://palaeo-electronica.org/paleo/2004_2/suture/issue2_04.htm>.
- Miller, A.K., Furnish, W.M. & Schidewolf, O.H. 1957: Paleozoic Ammonoidea. In Moore, R.C. (ed.): *Treatise on Invertebrate Paleontology, Volume L*, 11–79. Geological Society of America, Boulder, and University of Kansas Press, Lawrence.
- Olóriz, F. & Palmqvist, P. 1995: Sutural complexity and bathymetry in ammonites: fact or artifact? *Lethaia* 28, 167–170.
- Olóriz, F., Palmqvist, P. & Pérez-Claros, J.A. 1997: Shell features, main colonized environments, and fractal analysis of sutures in Late Jurassic ammonites. *Lethaia* 30, 191–204.
- Olóriz, F., Palmqvist, P. & Pérez-Claros, J.A. 2002: Morphostructural constraints and phylogenetic overprint on sutural frilling in Late Jurassic ammonites. *Lethaia* 35, 158–168.
- Pérez-Claros, J.A. 2005: Allometric and fractal exponents indicate a connection between metabolism and complex septa in ammonites. *Paleobiology* 31, 221–232.
- Pérez-Claros, J.A., Palmqvist, P. & Olóriz, F. 2002: First and second orders of suture complexity in ammonites: a new methodological approach using fractal analysis. *Mathematical Geology* 34, 323–343.
- Pérez-Claros, J.A., Olóriz, F. & Palmqvist, P. 2007: Sutural complexity in Late Jurassic ammonites and its relationship with phragmocone size and shape: a multidimensional approach using fractal analysis. *Lethaia* 40, 253–272.
- Saunders, W.B. 1995: The ammonoid suture problem: relationship between shell- and septal thickness and suture complexity in Paleozoic ammonoids. *Paleobiology* 21, 343–355.
- Saunders, W.B. & Work, D.M. 1996: Shell morphology and suture complexity in Upper Carboniferous ammonoids. *Paleobiology* 22, 189–218.
- Saunders, W.B. & Work, D.M. 1997: Evolution of shell morphology and suture complexity in Paleozoic prolecanitids, the rootstock of Mesozoic ammonoids. *Paleobiology* 23, 301–325.
- Saunders, W.B., Work, D.M. & Nikolaeva, S.V. 1999: Evolution of complexity in Paleozoic ammonoid sutures. *Science* 286, 760–763.
- Schidewolf, O.H. 1954: On development, evolution, and terminology of the ammonoid suture line. *Bulletin of the Museum of Comparative Zoology at Harvard College* 112, 217–237.
- Seilacher, A. 1975: Mechanische Simulation und funktionelle Evolution des Ammonite-Septems. *Paläontologische Zeitschrift* 49, 268–286.
- Seilacher, A. & LaBarbera, M. 1995: Ammonites as Cartesian divers. *Palaaios* 10, 493–506.
- Shevyrev, A.A. 2006: The cephalopod macrosystem: a historical review, present state of knowledge, and unsolved problem: 3. classification of Bacritoidea and Ammonoidea. *Paleontological Journal* 40, 151–160.
- Shigeta, Y. & Weitschat, W. 2004: Origin of the Ammonitina (Ammonoidea) inferred from the internal shell features. *Mitteilungen aus dem Geologisch-Paläontologisches Institut der Universität Hamburg* 88, 179–194.
- Tozer, E.T. 1981: Triassic Ammonoidea: classification, evolution and relationship with Permian and Jurassic forms. In House, M.R. & Senior, J.R. (eds): *The Ammonoidea. The Evolution, Classification, Mode of Life and Geological Usefulness of a Major Fossil Group*, 65–100. Academic Press, London.
- Ubukata, T. 2004: A three-dimensional digitizing system based on triangulation using multiple viewing images. *Geoscience Reports of Shizuoka University* 31, 65–72 [in Japanese with English abstract and figure captions].
- Ward, P.D. 1980: Comparative shell shape distributions in Jurassic–Cretaceous ammonites and Jurassic–Tertiary nautiloids. *Paleobiology* 6, 32–43.
- Weitschat, W. & Bandel, K. 1991: Organic components in phragmocones of Boreal Triassic ammonoids: implications for ammonoid biology. *Paläontologische Zeitschrift* 65, 269–303.
- Westermann, G.E.G. 1958: The significance of septa and sutures in Jurassic ammonite systematics. *Geological Magazine* 95, 441–455.
- Westermann, G.E.G. 1971: Form, structure and function of shell and siphuncle in coiled Mesozoic ammonoids. *Royal Ontario Museum, Life Sciences Contributions* 78, 1–39.
- Westermann, G.E.G. 1975: Model for origin, function and fabrication of fluted cephalopod septa. *Paläontologische Zeitschrift* 49, 235–253.

- Wiedmann, J. & Kullmann, J. 1981: Ammonoid sutures in ontogeny and phylogeny. In House, M.R. & Senior, J.R. (eds): *The Ammonoidea: the Evolution, Classification, Mode of Life and Geological Usefulness of a Major Fossil Group*, 215–255. Academic Press, New York.
- Wright, C.W., Callomon, J.H. & Howarth, M.K. 1996: Ammonoidea Volume 4. In Kaesler, R.L. (ed.): *Treatise on Invertebrate*

- Paleontology, Volume L*, Revised, 1–362. Geological Society of America, Boulder.
- Zahn, C.T. & Roskies, R.Z. 1972: Roskies, Fourier descriptors for plane closed curves. *IEEE Transactions, Computers C-21*, 269–281.